Entanglement of Atomic Ensembles by Trapping Correlated Photon States

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We describe a general technique that allows for an ideal transfer of quantum correlations between light fields and metastable states of matter. The technique is based on trapping quantum states of photons in coherently driven atomic media, in which the group velocity is adiabatically reduced to zero. We discuss possible applications such as quantum state memories, generation of squeezed atomic states, preparation of entangled atomic ensembles, quantum information processing, and quantum networking.

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One of the most intriguing aspects of quantum theory is the possibility to entangle quantum states of separated objects. Recently these ideas led to many interesting new concepts such as quantum cryptography [1], teleportation [2], and quantum computation [3]. Photons are the fastest, simplest, and very robust carriers of quantum information [4], but they are difficult to store.

This Letter describes a technique that allows one to transfer quantum correlations from traveling-wave light fields to collective atomic states and vice versa with nearly ideal efficiency. This is achieved by adiabatically reducing the group velocity of light to zero, thereby "trapping" the photons in the medium. Specifically, we use intracavity electromagnetically induced transparency (EIT) [5,6] in which the properties of a cavity filled with Λ-type atoms can be manipulated by an external (classical) field [7].

Once the transfer is completed, the atomic ensemble "stores" all photons including their quantum correlations in metastable many-atom states. Consequently, a procedure of this kind can be used to generate nonclassical states of atoms, and to entangle two or more ensembles by mapping entangled photon wave packets onto separated atomic systems. In addition to fundamental aspects, this makes applications in low-noise spectroscopy [8] and quantum teleportation of collective atomic states [2] feasible. Furthermore, the atomic excitations can be manipulated over a long period of time, which opens up interesting possibilities for information processing [3]. Finally, the stored quantum states can be transferred back to light by reversing the storage procedure.

The present contribution is motivated by recent experiments, in which EIT has been used to dramatically reduce the group velocity of light pulses [9,10]. Adiabatic passage has already been considered for manipulation of *single* atoms in the context of cavity QED [11]. In contrast, the method described here involves an optically dense *many-atom* system and does not require a strong-coupling regime. In the present system, single photons couple to *collective excitations* associated with EIT, and the corresponding coupling strength can exceed that of an individual atom by orders of magnitude. As opposed to approaches involving the partial transfer of correlations by dissipative means [12,13], the present method is state preserving, completely coherent, and reversible.

To illustrate the technique, consider a single-mode cavity filled with a large number of coherently driven Λ-type atoms as shown in Fig. 1a. One transition is coupled by the quantum cavity field, whereas the other is driven by a classical coherent field of Rabi frequency $\Omega(t)$. Under conditions of two-photon resonance, the driving field induces transparency [6] for the cavity field and the associated linear dispersion can substantially reduce its group velocity [9]. This leads to a dramatic enhancement of the effective storage time limited only by the lifetime of the dark state [5]. We note that narrow two-photon resonance can be achieved also in a Doppler-broadened medium if both fields have similar frequencies and are copropagating. In this case a ring cavity must be used. The Hamiltonian of the cavity + N-atom system can be written in terms of collective operators $\hat{\Sigma}_{ab} = \sum_{i=1}^{N} \hat{\sigma}^i_{ab}$ and $\hat{\Sigma}_{ac} = \sum_{i=1}^{N} \hat{\sigma}^i_{ac}$ as

$$H = \hbar g \hat{a} \hat{\Sigma}_{ab} + \hbar \Omega(t) \hat{\Sigma}_{ac} + \text{H.c.}, \qquad (1)$$

where $\hat{\sigma}^i_{\mu\nu} = |\mu\rangle_{ii}\langle\nu|$ is the flip operator of the ith atom between states $|\mu\rangle$ and $|\nu\rangle$. g is the coupling constant between the atoms and the field mode (vacuum Rabi

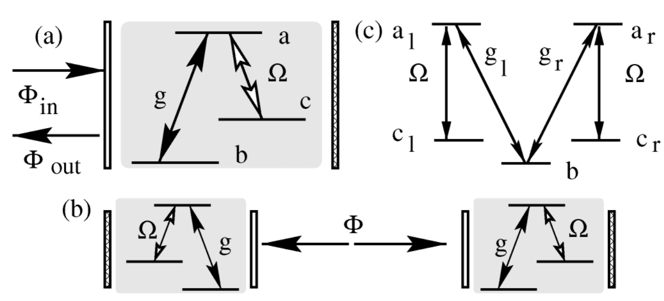

FIG. 1. (a) Optical cavity with single output mirror filled with large number of Λ-type atoms. External coherent field of Rabi frequency $\Omega(t)$ is used to control properties of resonator system. Input field state is transferred back and forth to atomic system via Raman adiabatic passage from states $|b\rangle$ to state $|c\rangle$. (b) Generation of entangled atomic ensembles using correlated photons. (c) Multistate atoms for trapping correlated photons of right and left circular polarizations.

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frequency) which for simplicity is assumed to be equal for all atoms. Here and below, we work in a frame rotating with optical frequencies. This Hamiltonian has a family of dark states that are decoupled from both optical fields:

$$|D,n\rangle \equiv \sum_{k=0}^{n} \sqrt{\frac{n!}{k!(n-k)!}} \frac{(-g)^k N^{k/2} \Omega^{n-k}}{(g^2N + \Omega^2)^{n/2}} \times |n-k\rangle |c^k\rangle .$$

They are composed of field states with $|n-k\rangle$ photons and symmetric Dicke-like atomic states $|c^k\rangle$ containing k atoms in level $|c\rangle$, and all others in the ground state $|b\rangle$:

$$|c^0\rangle \equiv |b_1 \cdots b_N\rangle, \qquad |c^1\rangle \equiv \sum_{i=1}^{N} \frac{-1}{\sqrt{N}} |b_1 \cdots c_i \cdots b_N\rangle,$$

$$|c^2\rangle \equiv \sum_{i \neq j=1}^{N} \frac{1}{\sqrt{2N(N-1)}} |b_1 \cdots c_i \cdots c_j \cdots b_N\rangle, \text{ etc.}$$

We here assumed that the number of atoms is much larger than the number of photons. The states $|D,n\rangle$ correspond to elementary excitations of bosonic quasiparticles, so-called dark-state polaritons [7].

The adiabatic transfer is based on the asymptotic behavior of the dark states in the two limiting cases:

$$|D,n\rangle \to |n\rangle |c^0\rangle, \quad \text{when } \Omega \gg g\sqrt{N}, \tag{2}$$

$$|D,n\rangle \to |0\rangle |c^n\rangle, \quad \text{when } \Omega \ll g\sqrt{N}. \tag{3}$$

For a sufficiently strong coherent driving field, the atoms do not interact with light, and the dark state coincides with the "bare" cavity mode and all atoms being in the ground state. In this limit photons can "leak" in and out of the cavity as if it would be empty. In the opposite limit, the dark state is a purely atomic state with no photons in the cavity. In this case the lifetime of excitations will not be sensitive to cavity decay; it will be limited solely by the decay of the metastable atomic states. It is important that by varying $\Omega(t)$, and, consequently, by changing the linear dispersion in the medium, the state of the combined atom + cavity system can be changed from cavitylike (in which excitation is mostly of photon nature) to atomlike (in which excitations are shared among the atoms). Since all dark states are orthogonal to each other, the ideal storing procedure (as discussed below) will transform any superposition of photon states into corresponding superpositions of atomic states:

$$\sum_i \alpha_i |i\rangle |c^0\rangle \to \sum_i \alpha_i |0\rangle |c^i\rangle . \tag{4}$$

Before proceeding with a detailed description of the technique, we note that the above results can be easily generalized to the case of two atomic ensembles, which can be entangled by trapping two entangled components of a photon field. In this case the atoms are placed either in the same or in two different optical cavities (Fig. 1b). The dark states are then the direct product of those corresponding to the subsystems: $|D,n,m\rangle = |D_r,n_r\rangle |D_l,m_l\rangle$. Hence, the following operation can be accomplished:

$$\sum_{nm} \alpha_{nm} |nm\rangle |c_r^0\rangle |c_l^0\rangle \to \sum_{nm} \alpha_{nm} |0\rangle |c_r^n\rangle |c_l^m\rangle . \tag{5}$$

It is clear that trapping perfectly entangled photon states will result in perfectly entangled atomic ensembles.

Yet another related situation involves trapping the states of two different field components within atoms of the *same* species. Here, the two fields interact with more complex atoms, such as those shown in Fig. 1c. By using essentially the same arguments as above, one finds that a perfect state transfer of the two fields to the atoms yields

$$\sum_{nm} \alpha_{nm} |nm\rangle |c_r^0 c_l^0\rangle \to \sum_{nm} \alpha_{nm} |0\rangle |c_r^n c_l^m\rangle . \tag{6}$$

The potential significance of the last scheme is that the correlations and the entanglement of the two fields can be manipulated, since they are now stored within the same atomic ensemble. This is of importance for information processing, in particular, for quantum logic devices.

We now describe and analyze the adiabatic procedure by which an input traveling-wave quantum field can be captured, stored, and released. To this end, we consider a quasi-1D system, include the continuum of the free-space plane-wave modes (with creation operators $b_k^\dagger$), and model the coupling of these modes to the cavity by an effective Hamiltonian $V = \hbar \sum_k \kappa \hat{a}^\dagger \hat{b}_k + \text{h.c.}$ κ is the coupling constant. The initial state of the free field is taken to be $|\Psi_{\text{in}}\rangle = \sum_k \xi_k^1 |1_k\rangle + \sum_{k,m} \xi_{k,m}^2 |1_k 1_m\rangle + \ldots$. It is convenient to work with correlation amplitudes, i.e., Fourier transforms of $\xi_{k\cdots l}^j$:

$$\Phi_j(t_1 \cdots t_j) \equiv \langle 0| \hat{E}(t_1) \cdots \hat{E}(t_j) |\Psi\rangle ,$$

where $\hat{E}(t) = (L/2\pi c) \int d\omega_k \exp(i\omega_k t) \hat{b}_k$, and L is the quantization length. For example, Φ_1 describes the envelope of a single photon wave packet, Φ_2 is the coincidence amplitude, etc. We now consider a broad class of pulsed fields described by a single envelope $h(t)$ such that

$$\Phi_j(t_1, t_2, \ldots, t_j) = \alpha_j \sqrt{j!}\, h(t_1) h(t_2) \cdots h(t_j) . \tag{7}$$

The quantum state of such pulses can be described by a rank-2 density matrix $\rho_{nm} = \alpha_n^* \alpha_m$. The corresponding mode function is a superposition of plane waves proportional to $h(z/c) = \int d\omega_k \, \xi_k e^{i\omega_k z/c}$. When the pulses interact with the combined system of cavity mode and atoms, the states $\alpha_j |c^j\rangle$ are excited. We proceed by deriving the equations of motion for the probability amplitudes in the basis of dark and orthogonal bright states. The bright states as well as the excited states (containing components of states $|a^i ..\rangle$) are then adiabatically eliminated. The remaining amplitudes of dark states and free-field components form a Dicke-like ladder. The ladder states are coupled to each other with the time-dependent coupling strength $\kappa \cos\theta(t)$, where $\cos\theta(t) = \Omega(t)/\sqrt{\Omega(t)^2 + g^2N}$.

In the case when only single-photon pulses are involved, the evolution equations are [14]

$$\dot{D}_1(t) = i\kappa \cos\theta(t) \sum_k \xi_k(t)\,, \tag{8}$$

$$\dot{\xi}_k(t) = -i\Delta_k\, \xi_k(t) + i\kappa \cos\theta(t) D(t)\,. \tag{9}$$

We proceed by formally integrating Eq. (9), substituting the result into Eq. (8), and invoking a Markov approximation. Assuming that no photons arrive to the cavity before t_0, we find for the dark-state amplitude $D_1(t) = -i\alpha_1 d(t)$ with

$$d(t) = \sqrt{\gamma \frac{c}{L}} \int_{t_0}^{t} d\tau \cos\theta(\tau) h(\tau) \times \exp\left\{-\frac{\gamma}{2}\int_{\tau}^{t} d\tau' \cos^2\theta(\tau')\right\}, \tag{10}$$

and for the input-output relation,

$$h_{\rm out}(t) = h(t) - \sqrt{\gamma L/c}\, d(t)\,, \tag{11}$$

where $h_{\rm out}(t)$ is the pulse shape of the outgoing wave packet. Here we have introduced the empty-cavity decay rate $\gamma = \kappa^2 L/c$. In order to trap photons, we require $h_{\rm out}(t) = \dot{h}_{\rm out}(t) = 0$. Differentiating Eqs. (10) and (11) yields

$$-\frac{d}{dt}\ln\cos\theta(t) + \frac{d}{dt}\ln h(t) = \frac{\gamma}{2}\cos^2\theta(t)\,. \tag{12}$$

If $\Omega(t)$ is chosen such that $\cos\theta(t)$ obeys Eq. (12) with the asymptotic condition $\cos\theta \to 0$, the output field remains zero. The above condition corresponds to a dynamical impedance matching [14]. The term on the right-hand side is the effective cavity decay rate reduced due to intracavity EIT [5]. The first term on the left-hand side of Eq. (12) describes internal "losses" due to coherent Raman adiabatic passage and the second term is due to the time dependence of the input field. As in the case of classical impedance matching [15], Eq. (12) reflects the condition for complete destructive interference resulting in a vanishing outgoing wave. Solving Eq. (12) yields

$$\cos^2\theta(t) = \frac{h^2(t)}{\gamma \int_{-\infty}^{t} d\tau\, h^2(\tau)}\,, \tag{13}$$

which corresponds to $d(t\to+\infty)\to 1$ (see Fig. 2a). Hence, by suitable variation of the classical driving field any single-photon pulse can be trapped ideally, if its pulse length is longer than the bare-cavity decay time.

Generalizations to multiphoton states can proceed along the same lines, but involve more tedious algebra. In particular, for the two-photon states one finds $D_2(t) = -\alpha_2 d(t)^2$, and, in general,

$$D_k(t) = (-i)^k \alpha_k d(t)^k \tag{14}$$

can be proved. Under conditions of quantum impedance matching $D_k(t\to\infty)\to(-i)^k\alpha_k$ for arbitrary k. Hence,

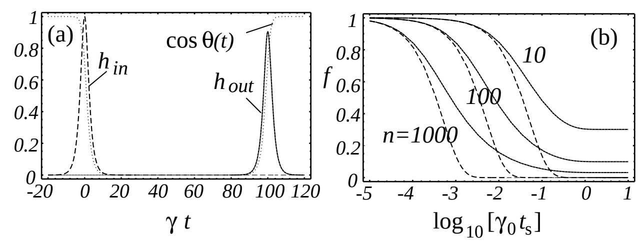

FIG. 2. (a) Storage of a hyperbolic secant pulse. Shown are normalized input (dashed line) and output pulse (full line) as a function of time as well as time dependence of $\cos\theta(t)$, optimized for the input field. Time unit is decay time of bare cavity γ^{-1}. Dark-state decay rate is $\gamma_0 = 10^{-3}\gamma$. (b) Fidelity of storage for Fock state (dashed line) and squeezed vacuum state (solid line) inputs as function of storage time t_s. n denotes mean number of photons.

pulsed fields in an arbitrary quantum state of a generalized single mode can be mapped to the atomic ensemble.

Releasing the stored quantum state into a pulse of desired shape can be accomplished in a straightforward way. A simple reversal of the time dependence of the control field at a later time t_d leads to a perfect mirror image of the initial pulse. However, $\cos\theta$ can also be rotated back to its original value in another way, which allows one to "tailor" the pulse shape of the outgoing wave packet while retaining the quantum state.

We next examine the factors limiting the performance of the photon trapping scheme. Adiabatic following occurs if the population in the excited and bright states is small at all times. The corresponding conditions can be derived by substituting the adiabatic solutions into the exact equations and requiring that the coupling of dark states to all other states is small [16]. For a pulse duration T, a linewidth of the excited state γ_a, and a cavity width γ, the adiabaticity conditions are

$$\Omega(t)^2 + g^2 N \gg \max\left[\gamma\gamma_a,\ \frac{\gamma_a}{T},\ \sqrt{\frac{\gamma}{T}}\,\gamma_a\right]. \tag{15}$$

Since the characteristic input-pulse length and thus the characteristic times T have to be larger or equal to the bare-cavity decay time γ^{-1}, the first condition is the most stringent one. Therefore adiabatic following is possible provided that $g^2 N \gg \gamma\gamma_a$.

In the discussion above, we have disregarded the finite lifetime of the metastable state, γ_0^{-1}. If γ_0 is small, its influence during the loading and unloading periods can be neglected but needs to be taken into account during the storage interval. Collective states such as $|c^n\rangle$ will dephase at a rate $\gamma_n = n\gamma_0$, which sets the upper limit on the longest storage time. To illustrate the effect of this damping, we have plotted in Fig. 2b the fidelity of the quantum state storage, defined as $f = {\rm Tr}\{\rho_{\rm in}\rho_{\rm out}\}$, as function of the storage time t_s for input pulses in a number state and a squeezed vacuum state. It is apparent that the maximum storage time is on the order of the single-atom decay time divided by the characteristic number of input photons. We note that in alkali-vapor cells with buffer gas and/or wall coatings dark-state lifetimes on the order of seconds are observed [9].

We conclude by summarizing the main results and outlining the possible avenues of future studies opened by this work. We demonstrated that it is possible to map ideally the quantum states of light fields onto metastable states of atomic ensembles. This allows for the generation of nonclassical (e.g., squeezed) states of atoms [see Eq. (4)]. These states are precisely of the form required to achieve spectroscopic sensitivity beyond the usual quantum limit [8]. We have further shown that by trapping entangled fields it is possible to generate entangled atomic ensembles [see Eq. (5)]. Quantum teleportation of collective atomic states and quantum networking is, hence, feasible. Finally, by trapping two fields within the same multistate species [see Eq. (6)] one can create good conditions for coherent manipulation of field correlations, entanglement, etc. This opens up interesting prospectives for quantum logic operations.

It is essential that all of the transfer operations can be achieved without invoking the strong coupling regime of single-atom cavity QED. We have shown that under conditions of quantum impedance matching free fields can be ideally transferred back and forth provided the excitation rate of the collective mode ($\sim g^2N/\gamma_a$) exceeds the cavity decay rate [see Eq. (15)]. This can be used to considerably improve the fidelity of quantum processing.

In proof-of-principle experiments squeezed or EPR-correlated light generated by optical parametric oscillators [17] can be used to drive dark resonances in Cs atoms. However, eventually it should be possible to use the nonclassical fields generated by EIT-based nonlinear processes [18] in the medium that is itself used for trapping.

We note that several interesting questions remain open and need to be explored. For example, we have not considered here any specific schemes to perform quantum logic gates with trapped photons. Possible ways include cavity QED techniques [11], direct nonlinear interactions of photons via, e.g., resonantly enhanced Kerr nonlinearities [18], or, alternatively, atom-atom interactions. We further note that although the present analysis involves 7467. electronic degrees of freedom, our method can be used to excite states of the center-of-mass motion of cold atomic samples in Bose-Einstein condensates [19]. Here again collisions need to be taken into account. This adds new interesting dimensions to the present studies and will discussed elsewhere.

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