A Smidgin of Quantum Optics

Photon Bunching, Quantum Light, and Slow Light

http://quoptics.nusoy.com

Yosun Chang

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Abstract: Some interesting aspects of quantum optics are reviewed at a level suitable to the audience of an introductory course on Solid State Physics (Physics 211A at UCSD). Knowledge of Quantum Mechanics at the level of Sakurai is assumed. An Appendix broaches up field quantization, coherent states, and both the semiclassical and quantum electrodynamical Rabi Model. I have taken the liberty to create my own diagrams and rewrite things from my own perspective; but, if I am not error-free, I would appreciate your comments through email via *yosun@nusoy.com*.

1 Classical Light: Photon Bunching

In the 1950s, Hanbury, Brown and Twiss discovered that classical light sources, such as thermal or coherent states, display the phenomenon of *photon bunching*. That is, photons tend to arrive in pairs, rather than individually. In honor of this seminal discovery, the effect is called the *Hanbury Brown Twiss Effect*. Their experiment is described in the diagram below (Figure 1).

A beam of photons is sent into a 50-50 beam-splitter. Detectors D1 and D2 are placed equidistance from the beam-splitter. The path through D1 experiences a variable time-delay t that is less than the coherence time. The two beams meet at the coincidence counter (CC). It is found that for vanishing time-delay, the intensity of photons detected at the CC is twice as great. This suggests that photons arrive in pairs—a bunch.

Recall that the early (1900s) interference experiments were done through the light source of a gas flame, or perhaps, another variant still within the realm of a thermal source. At the alleged single-photon limit, interference effects were observed. The conclusion of such experiments, to quote Dirac, is that "each photon interferes only with itself; interference between two photons does not occur." After the invention of the laser, coherent light sources were used to arrive at the same interference patterns. But, as shown in the footnote, both thermal sources and coherent sources are prone to photon bunching.[1]

Aside: A physicist,[2] well-known in the field of quantum computing, subscribes to a multiverse theory wherein he justifies the existence of the multiverse

[1] A coherence function with the argument of the time-delay, $g(\tau)$, can be derived from calculating the intensity based on a quantized field (see the Appendix), keeping only the field term with the creation operator $a^\dagger$. A coherence greater than 1 means twice the intensity of single photons. The results are as follows:

1. $g^{(2)}(\tau) = 2$ for a single-mode thermal state
2. $g^{(2)}(\tau) \in [1, 2]$ for a multi-mode thermal state
3. $g^{(2)}(\tau) = 1$ for a multi-mode coherent state
4. $g^{(2)}(0) < g^{(2)}(\tau)$ for *antibunched photons*

Immediately, one sees that both thermal states are prone to photon bunching. The multi-mode coherent states also show the same effect since the photons arrive randomly, as per the Poisson Distribution (see Appendix). Thus, even when attenuated, the coherent sources might fire more than one photon.

[2] D. Deutsch. *The Fabric of Reality*

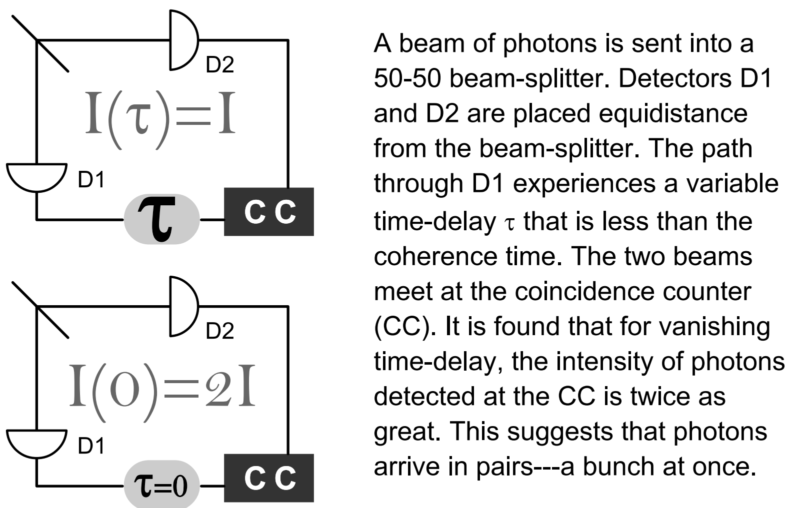


Figure 1: The Hanbury Brown Twiss Experiment, a.k.a., Photon Bunching Effect.

via *shadow photons*. He arrives at this conclusion from interference experiments, stating that even when the light intensity is brought down to a minimum extremity, such that only a single photon is transmitted, interference still exists. He claims the only way this is possible is if the (real) photons arrive after interaction with virtual photons, which he dubs the name *shadow photons*. However, because photons from both coherent and thermal sources arrive in *pairs*, his shadow photons are *much* too ad hoc to justify a theory.[3]

To summarize, a photon displays a wave nature in interference experiments because it interacts with another photon, viz., the mechanism of *photon bunching*. Photons, as if water molecules, tend to stick together. In contrast, the next section describes quantum photons—the ones that are true quanta.

2 Quantum Light

Quantum light consists of *antibunched photons*. Such photons do *not* show interference effects in interference experiments. However, given multiple paths to choose from, the wave-nature of photons can be re-attained; antibunched photons can be made to interfere through, for example, a Mach-Zehnder Interferometer.

[3]Although I, too, ascribe to the multiverse theory, I arrive at the conclusion via a different means—the quantum brain. Don't worry. I won't pursue the exploration of my belief in full (if I continue in this field) until after I've achieved something truly notable. Maybe I'd pull off a Josephson.

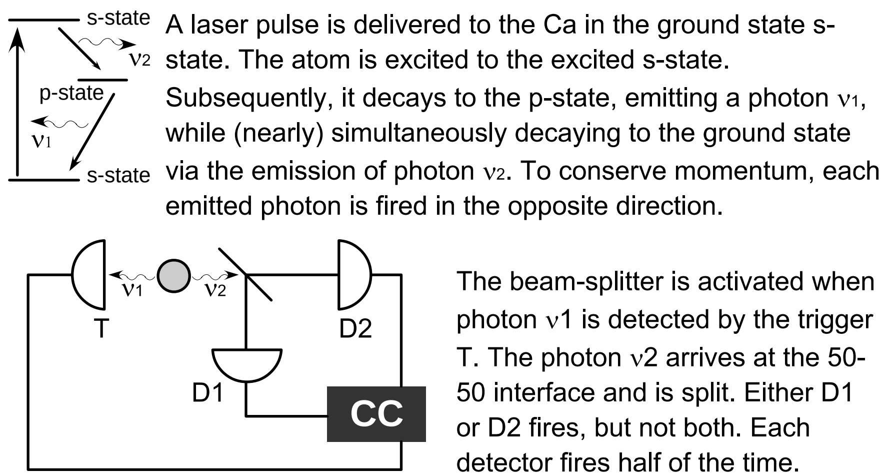


Figure 2: Grangier, et al., single antibunched photon source.

Grangier [2], et al, devised a source of antibunched photons. A Calcium atom is excited by a laser pulse. As the atom decays from the excited state, a single photon is emitted. The beam-splitter apparatus shown in the figure confirms that a single photon is emitted (Figure 2).

3 Slow Light

Slow light phenomena have been observed in processes involving Electromagnetically Induced Transparency (EIT). [3], [4]

Electromagnetically Induced Transparency (EIT) is, simply stated, the lossless transmission of a signal light pulse (through an atomic medium) that is induced via a second driven light pulse. However, because the signal propagates without loss, it is spatially compressed and its group velocity is reduced. Thus, arises the phenomenon of slow light.

This section begins with a review of the EIT theory, then continues with various slow light experiments, concluding with a practical application that has recently been experimentally realized (November 2005).

3.1 Electromagnetically Induced Transparency (EIT)

In 1991, Harris, et al., showed that one can “turn off" absorption of a signal pulse by using another laser pulse [5].

Consider a three-level atomic system. The ground state consists of two hyperfine levels, $|1\rangle$ and $|2\rangle$, which can be excited to the top state $|3\rangle$ by the absorption of a photon. Without the interaction of a *control field*, i.e., a second laser that is driven at the frequency ω_c, the incoming signal pulse (probe) of ω_p would be absorbed, resulting in lossy transmission.

However, if the control laser ω_c is tuned to the transmission between $|2\rangle$ and $|3\rangle$ and the probe laser ω_p is tuned to the transmission between $|1\rangle$ and $|3\rangle$, a coherent superposition of $|1\rangle$ and $|2\rangle$ states is produced. The eigenstates of the Hamiltonian change, and the system becomes that of *the field and the atom*. The new eigenstates are the two coherent superposition of ground state hyperfine states, $|NC\rangle$ and $|C\rangle$, and the excited state $|3\rangle$ remains an eigenstate.

The coherent superposition of states of the new Hamiltonian yields *polaritons*. A Raman transition couples the light to the coherent states. Because the hyperfine states, $|1\rangle$ and $|2\rangle$, make equal and opposite contributions, destructive interference nullifies the transition dipole moment between $|NC\rangle$ and $|3\rangle$; as dark as dark can be, the polaritons are thus called *dark state polaritons*. Thus, the non-coupled $|NC\rangle$ state is decoupled from the excited state $|3\rangle$, which prevents the absorption of light, allowing for loss-less transmission. (One can neglect $|C\rangle$, the state coupled to $|3\rangle$, since it is unpopulated.)

3.1.1 Some Math

The $|2\rangle$ to $|3\rangle$ transition can be approximated by a classical field of Rabi frequency $\Omega(t) = \omega_c$, while the $|1\rangle$ to $|3\rangle$ transition (of frequency ω_p) should be approximated by a quantum field, whose derivation is sketched earlier in the Appendix,

$$\hat{E}(z,t) = \sum_k a_k(t) e^{ikz} e^{-i(\omega_p/c)(z-ct)}. \tag{1}$$

Quantum properties of the medium are described by the collective slow-varying atomic operators averaged over N_z, the number of particles at position z,

$$\hat{\sigma}_{\alpha\beta}(z,t) = \frac{1}{N_z} \sum_{j=1}^{N_j} |\alpha_j\rangle\langle\beta_j| e^{-i\omega_{\alpha\beta} t}, \tag{2}$$

where the subscript $\alpha\beta$ denotes the transition from $|\alpha\rangle$ to $|\beta\rangle$.

The evolution of $\hat{E}(z,t)$ corresponding to the probe pulse is described, in a slowly varying amplitude approximation, by the propagation equation,

$$\left(\frac{\partial}{\partial t} + c\frac{\partial}{\partial z}\right) \hat{E}(z,t) = igN\hat{\sigma}_{\alpha\beta}(z,t), \tag{3}$$

where $g = p\sqrt{\frac{\omega_p}{2\hbar\epsilon_0 V}}$ and V is the quantization volume, while p is the dipole moment of the $|1\rangle$ to $|3\rangle$ transition.

In the perturbative and adiabatic limit, the propagation of the quantum pulse is governed by this equation,

$$\left(\frac{\partial}{\partial t} + c\frac{\partial}{\partial z}\right) \hat{E}(z,t) = -\frac{g^2 N}{\Omega(t)} \frac{\partial}{\partial t} \frac{\hat{E}(z,t)}{\Omega(t)}, \tag{4}$$

where N is the number of atoms in the volume.

The solution can be obtained by introducing a new quantum field via the canonical transformation,

$$\hat{\Psi}(z,t) = \cos\theta(t)\hat{E}(z,t) - \sin\theta(t)(N)^{1/2}\hat{\sigma}_{12}(z,t), \tag{5}$$

where $\cos\theta(t) = \frac{\Omega(t)}{(\Omega^2(t)+g^2N)}^{1/2}$ and $\sin\theta(t) = \frac{g\sqrt{N}}{\Omega^2(t)+g^2N}$.

The group velocity v_g is given by $v_g = c\cos^2\theta(t)$. The equation predicts that as the group velocity decreases as the control frequency decreases, since $\Omega(t) = \omega_c \to 0 \Rightarrow \cos\theta(t) \to 0$. Light can be stopped when the control intensity is 0 ($\omega_c = 0$).

This is a summary of equations presented in [6].

3.2 Slow Light in Cold Atoms

Experimentally, one turns on the control field ω_c before propagating the probe pulse ω_p. Initially, only the ground state $|1\rangle$ is populated. When the control field is turned on, the $|2\rangle$ state is coupled to the $|3\rangle$ state at a frequency of ω_c; however, because $|2\rangle$ is unpopulated, $|3\rangle$ splits into two hyperfine states of frequency difference $\Delta\nu$. The intensity of the control (coupling) laser is proportional to the square of this difference, $I \propto \Delta\nu^2$.

Enter the probe pulse: because the contribution to susceptibility from the $|3\rangle$ hyperfine states exactly cancels when the probe frequency ω_p is at resonance with the $|1\rangle$ to $|3\rangle$ transition, the index of refraction is exactly 1. This produces a steep slope in the index of refraction, which decimates the group velocity, since the group velocity v_g is related to the index of refraction slope n_g by $v_g \propto 1/n_g$.[4] (Figure 3)

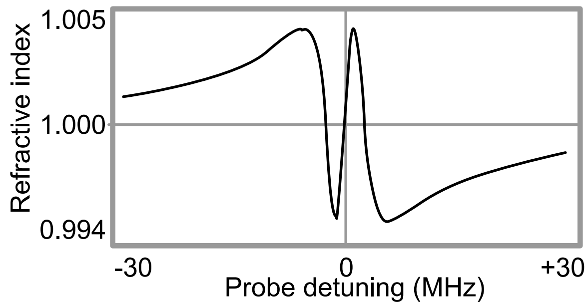
Note that because $n \approx 1$ (the probe pulse is tuned to the $|1\rangle$ to $|3\rangle$ resonance), the phase velocity $v_p = c$ is just the speed of light in vacuum. However, because a probe pulse contains multiple Fourier components at varying frequencies, the sum of v_p variations for each frequency results in an envelope that travels much slower than the individual frequency components—hence, the decrease of the group velocity v_g. (See *http://quoptics.nusoy.com* for a neat animation of this effect.)

3.2.1 Hau et al. (1999)

Hau [7] used Na atoms (density $10^{12} - 10^{14}$ $atoms/cm^3$) cooled to a sub-Bose-Einstein condensation temperature $T < T_c = 435nK$.[5] The experimental setup is as follows:

[4] Aside: The full expression for the group velocity is $v_g = Re\left(\frac{d\omega}{dk}\right) = v_f - v_s$, where $v_f = Re\left(\frac{c}{n(\omega,k)+\omega\frac{\partial n(\omega,k)}{\partial\omega}}\right)$ is due to frequency dispersion and $v_s = Re\left(\frac{\omega\frac{\partial n}{\partial k}}{n(\omega,k)+\omega\frac{\partial n(\omega,k)}{\partial\omega}}\right)$ is due to spatial dispersion. At resonance, $\frac{dn}{dk} \to 0$, and thus $v_g = v_f$. Thus, for, large $\frac{dn}{d\omega}$, the group velocity becomes small. (It is useful to define $n_g = n(\omega,k) + \omega\frac{\partial n(\omega,k)}{\partial\omega}$.)

[5] The coolest temperature was $50nK$.



Note that the plot is centered at n=1.000. The slope near the origin becomes increasingly steep, indicative of the decrease in group velocity.

Figure 3: Plot of index of refraction for slow light experiment.

The control pulse (coupling beam), linearly polarized, is injected through the $y - axis$ (width) of the atomic sample. A circularly polarized probe pulse is then injected through the $z - axis$ (length) of the sample. A photo-multiplier tube (PMT) is used to measure the delay and transmission of the probe pulse. The $z - x$ extent of the atomic cloud is determined via a third laser and a CCD camera (CCD2), while the $x - y$ extent is determined via CCD1. (Figure 4).

One advantage of using very cold atoms is that there is negligible Doppler smearing of the hyperfine energy levels, and thus a low coupling intensity is possible. As the intensity of the control laser (coupling intensity) is lowered, the slope of the index of refraction becomes steeper, and the group velocity decreases.

3.3 Slow Light in Hot Atoms

Hot atoms (at the regime of room temperature $T \approx 300K$) are prone to Doppler smearing. Because the thermal energy of the atoms is great, the control laser intensity must be compensatingly great to hold the atoms. The group velocity thus remains big.

However, if the probe and control pulses are driven at equal frequencies (and are collinear), Doppler smearing, which is due to the difference between ω_p and ω_c, can be avoided.

3.3.1 Budker et al. (1998)

Budker [8] used Rb vapor (density 10^{12} $atoms/cm^3$) at room temperature to slow light to $v_g = 8m/s$.

3.4 Stopped Light!

Recall that the group velocity of the probe pulse is related by $v_g \frac{d\omega}{dn}$. By lowering the intensity of the control beam, the slope of the index of refraction $\frac{dn}{d\omega}$

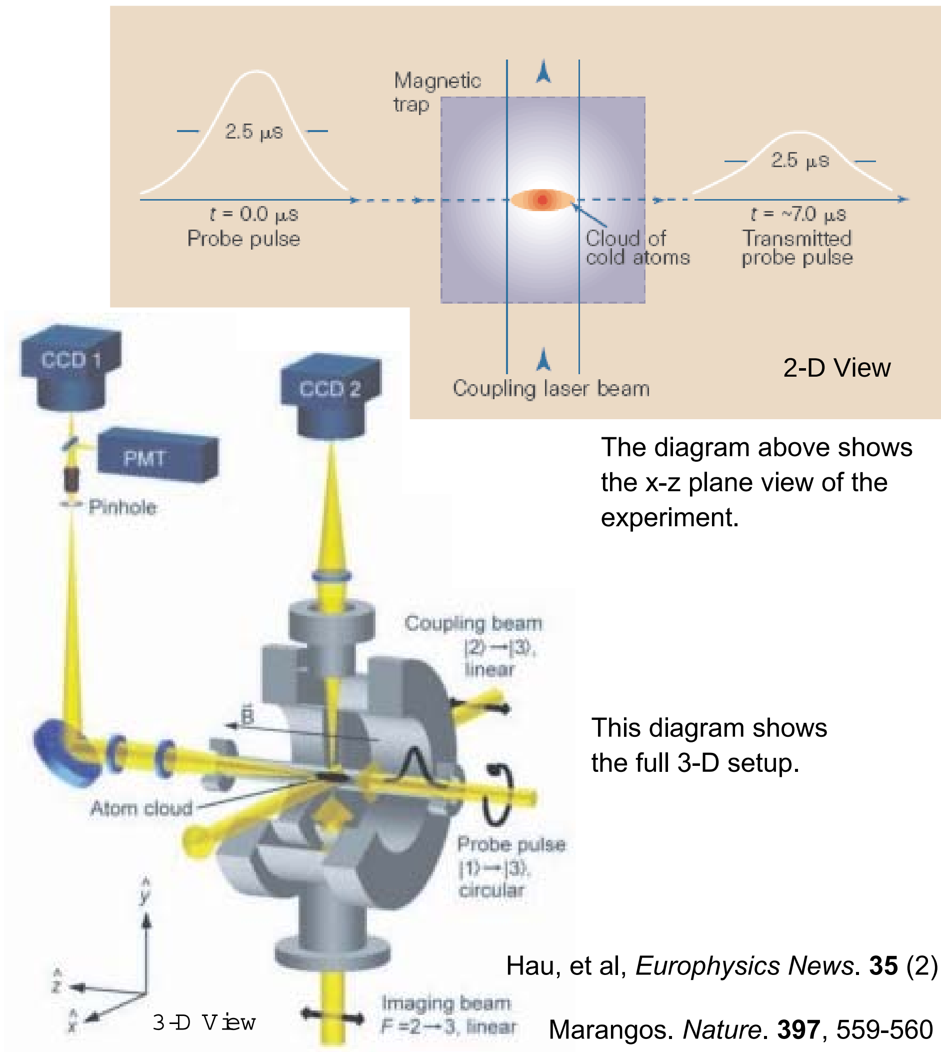


Figure 4: Hau 1999—a basic setup for a slow light experiment

increases. Turning off the control beam suddenly produces an infinite slope in the index of refraction; lo and behold, v_g 0, light is stopped.

3.5 Information Storage

When the control field (coupling field) is turned off smoothly, the dark state polariton is adiabatically converted to a purely atomic excitation. This allows for data storage, with the input being the incoming probe pulse. When the control field is slowly turned on, the dark state polariton is adiabatically converted to the transmitted probe pulse, i.e., the output.[6] [9]

3.5.1 Vlasov, et al. (2005)

Vlasov [10] has achieved the active control of slow light. This is a monumental step in data storage, since now the stored qubit may be actively manipulated. The atomic medium consists of *silicon photonic waveguides* with resonant photonic structures formed by etching periodic arrays of holes in Si-suspended membranes. A single-mode access strip waveguide couples the light to the photonic waveguide.

For accurate measurement of the group velocity, an integrated Mach-Zehnder interferometer was used.

4 Envoi

The purpose of this paper serves to inform the reader of the author's more interesting finds while reading up on quantum optics for the aforementioned course. The findings perplex the author, who leaves unanswered the following questions:

1. Since photons arrive in pairs, does this mean that neutrons and electrons also arrive in pairs, as in neutron or electron diffraction/interference?[7]

2. What happens if the slow light experiments were done with a source of antibunched photons?

[6] The conversion of the light to atomic excitation effectively stops it. It is interesting that qubits in the future might be based on cores of frozen light.

[7] Diffraction and interference are basically the same thing; they both involve superpositions of coherent waves of different phase.

A Appendix

This section reviews field quantization, coherent states, and the Rabi Model.

A.1 Field Quantization

Field quantization[8] in a single sentence: Starting with the free-space sans-source Maxwell's equations,

$$\nabla \cdot \vec{E} = 0 \qquad \nabla \times \vec{E} = \frac{\partial \vec{B}}{\partial t} \tag{6}$$

$$\nabla \cdot \vec{B} = 0 \qquad \nabla \times \vec{B} = \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} \tag{7}$$

and the single-mode fields,[9]

$$E_x(z,t) = \left(\frac{2\omega^2}{V\epsilon_0}\right)^{1/2} q(t)\sin(kz) = E_0 q \sin(kz) \tag{8}$$

$$B_y(z,t) = \left(\frac{\mu_0\epsilon_0}{K}\right)\left(\frac{2\omega^2}{V\epsilon_0}\right)^{1/2} \dot{q}(t)\cos(kz) = B_0 p \cos(kz), \tag{9}$$

one observes that the form of the field Hamiltonian,

$$\frac{1}{2}\int dV \left(\epsilon_0 \vec{E}^2 + \frac{1}{\mu_0}\vec{B}^2\right) \tag{10}$$

has the same form as the simple harmonic oscillator Hamiltonian,

$$\frac{1}{2}\left(\hat{q}^2\omega^2 + \hat{p}^2\right); \tag{11}$$

this inspires the re-expression of the canonical position and momentum[10] as $\hat{q} = \frac{1}{2\omega}\left(\hat{a} + \hat{a}^\dagger\right)(2\hbar\omega)^{1/2}$ and $\dot{q} = \hat{p} = \frac{1}{2i}\left(\hat{a} + \hat{a}^\dagger\right)(2\hbar\omega)^{1/2}$, which quantizes the field according to the simple harmonic oscillator annihilation (lowering a) and creation (raising $a^\dagger$) operators,[11]

$$E_x(z,t) = \mathcal{E}_0 \hat{q}\sin(kz) \tag{12}$$

$$B_y(z,t) = \mathcal{B}_0 \hat{p}\cos(kz) \tag{13}$$

In the above, Second Quantization or Canonical Quantization is used to present an elementary discussion of the topic.

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[8] The most elementary example of a field confined to a one-dimensional (z) cavity is shown, which can easily be generalized to multi-mode fields.

[9] V is the effective volume of the cavity, while $k = \omega/c = \sqrt{\mu_0\epsilon_0}\omega$ is the wave number of the mode, and the other constants have their usual meaning.

[10] Unit mass employed.

[11] $\mathcal{E}_0 = \left(\frac{\hbar\omega}{V\epsilon_0}\right)^{1/2}$ and $\mathcal{B}_0 = \frac{\mu_0}{k}\left(\frac{\epsilon_0\hbar\omega^3}{V}\right)^{1/2}$ represent field per photon.

A.2 Coherent States

The state dubbed the name *coherent state* is the most classical quantum state of the simple harmonic oscillator. For one, the probability weighting functions $P(\alpha)$ of a coherent state are all $P(\alpha) \geq 1$—which is quite in sync with classical probability. There exist quantum states, such as squeezed light,[12] wherein $P(\alpha) < 1$.[13]

One can arrive at the expression for a coherent state from the following ansatz:[14]

$$\begin{cases} a|\alpha\rangle = \alpha|\alpha\rangle \\ |\alpha\rangle = \sum_n c_n|n\rangle \end{cases} \tag{14}$$

Normalizing, one has the coherent state $|\alpha\rangle$,[15]

$$|\alpha\rangle = e^{-\frac{1}{2}|\alpha|^2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}}|n\rangle. \tag{15}$$

One can calculate the expectation value of the electric field,[16]

$$\langle\alpha|\hat{E}_x(\hat{r},t)|\alpha\rangle = 2|\alpha| \left(\frac{\hbar\omega}{2\epsilon_0 V}\right)^{1/2} \sin(\omega t - \vec{k}\cdot\vec{r} - \theta), \tag{16}$$

which resembles a classical field, thereby further justifying the coherent state as the most classical state.

Coherent states are essentially Poissonian, since $\langle\alpha|\hat{n}^2|\alpha\rangle = (\bar{n})^{1/2}$ and $P_n = |\langle n|\alpha\rangle|^2 = e^{-|\alpha|^2}\frac{|\alpha|^{2n}}{n!} = e^{-\bar{n}}\frac{\bar{n}^n}{n!}$. Both deductions are easy exercises the reader ought to do.

The uncertainty or the fluctuation of the field is given by,[17]

$$\Delta E_x = \left(\langle E_x^2\rangle - \langle E_x\rangle^2\right)^{1/2} = \left(\frac{\hbar\omega}{2\epsilon_0 V}\right)^{1/2} \tag{17}$$

Interestingly, this yields a "thicker-lined" phase-space curve for the coherent state. Note that for a coherent state, the line is everwhere thicker, which contrasts with a squeezed state, where the line is thicker in some places and thinner in others.[18] The phase space diagrams (5) concludes this brief exposition to coherent states. (For more, see [1], Chapter 3ff.)

[12] Amplitude or quadrature squeezing, that is.

[13] Since you set a page limit for this paper, instead of enlightening you with the full details, I shall leave it to your imagination and not exceed the count.

[14] This is the standard way. I prefer defining a displacement operator, $\hat{D}(\alpha) = e^{\alpha a^\dagger - \alpha^* a}$, which acts on the vacuum state $|0\rangle$ to produce $|\alpha\rangle = \hat{D}(\alpha)|0\rangle$. The calculation is trivial via the *disentangling theorem*, $e^{A+B} = e^A e^B e^{-\frac{1}{2}[A,B]} = e^B e^A e^{\frac{1}{2}[A,B]}$, where $[A,B] \neq 0$ and $[A,[A,B]] = [B,[A,B]] = 0$.

[15] n is the n$\underline{^{th}}$ energy eigenstate, while $\hat{n} = a^\dagger a$ is the number operator.

[16] The Heisenberg Equations gives the time-dependence of the a operators, i.e., $\hat{a}(t) = \hat{a}(0)e^{i\omega t - ikx}$.

[17] V is the volume, and the other terms have their usual meaning.

[18] Imagine squeezing a water balloon. The volume becomes fat in some places, and thin in other places.

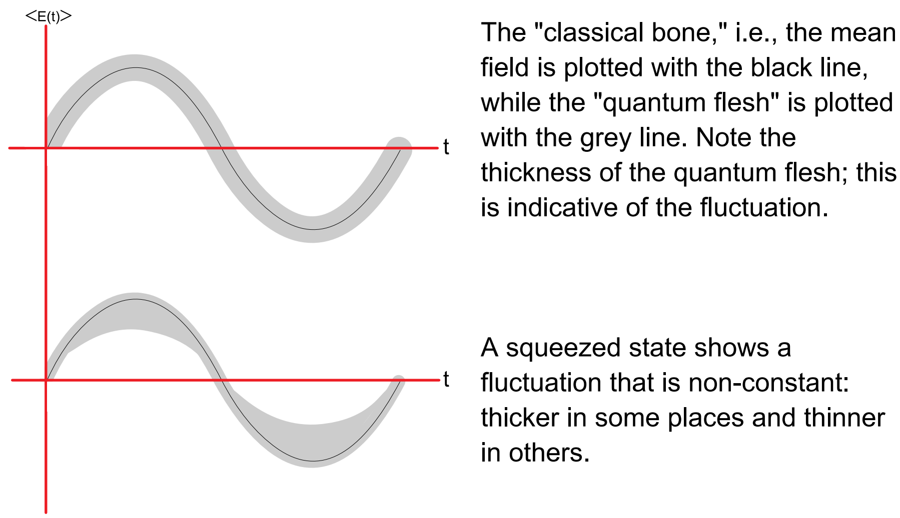

Figure 5: The plots show the "quantum flesh on classical bones." (To quote Gerry and Knight [1].)

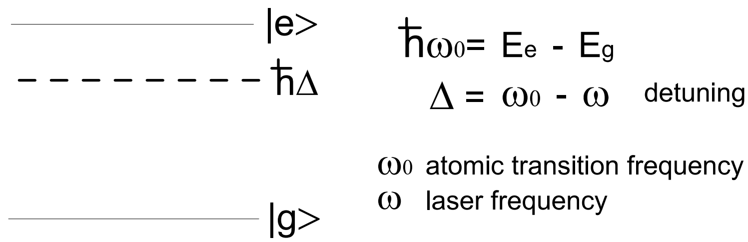

Figure 6: The Rabi energy levels and some parameters.

B The Rabi Model

Suppose a strong laser field driven at a frequency ω near the atomic transition frequency ω_0 interacts with the atomic system. (See Figure 6) This causes the population to transfer to the nearest resonant state, and thus only the dominant states are retained—namely, the ground state $|g\rangle$ and the excited state $|e\rangle$. This is the Rabi Model.[19]

[19] It effectively reduces complicated atomic systems to just a two-level system.

B.1 The Semiclassical Rabi Model

In the semiclassical model, the field is not quantized, and thus the Hamiltonian is simply,[20]

$$\hat{H}(t) = \hat{H}_0 + \hat{V}_0 \cos(\omega t), \tag{18}$$

where $\hat{V}_0 = -\vec{d} \cdot \vec{E}_0$ and $\vec{d}$ is just the dipole matrix.

The state vector is given by,

$$|\Psi(t)\rangle = c_g(t) e^{-iE_g t/\hbar} |g\rangle + c_e(t) e^{-iE_e t/\hbar} |e\rangle, \tag{19}$$

where $|e\rangle$ is the excited state and $|g\rangle$ is the ground state.

Plugging the above into the time-dependent Schrödinger equation, and making the Rotating Wave Approximation, one arrives at

$$c_e(t) = \frac{i\nu}{\Omega_R \hbar} e^{i\Delta t/2} \sin(\Omega_R t/2) \tag{20}$$

$$c_g(t) = e^{i\Delta t/2} \left(\cos(\Omega_R t/2) - i\frac{\Delta}{\Omega_R} \sin(\Omega_R t/2) \right), \tag{21}$$

where $\nu = \langle e|V_0|g\rangle = -d_{eg} \cdot E_0$ and $\boxed{\Omega_R = \left(\Delta^2 + \nu^2/\hbar^2\right)^{1/2}}$ is the (semiclassical) *Rabi frequency*.[21]

Thus, the probabilities for the excited and ground states are, respectively,

$$P_e(t) = |c_e(t)|^2 = \frac{\nu^2}{\Omega_R^2 \hbar^2} \sin^2(\Omega_R t/2) \tag{22}$$

$$P_g(t) = |c_g(t)|^2 = \cos^2(\Omega_R t/2) + \frac{\Delta^2}{\Omega_R^2} \sin^2(\Omega_R t/2) \tag{23}$$

At resonance, the detuning vanishes, $\Delta = 0$, and the probability for the ground state is just $P_g(t, \Delta = 0) = \cos^2(\Omega_R t/2)$.

It is useful to define an atomic inversion equation by $W(t) = P_e(t) - P_g(t)$, which in the case of $\Delta = 0$ becomes $W(t, \Delta = 0) = -\cos(\nu t/\hbar)$.

The equations suggest that the system continuously oscillates (in time) from the ground state to the excited state. For the case of $\Delta = 0$, at $t = \pi\hbar/\nu$ the entire population is excited.

B.2 The QED Rabi Model: The JC Hamiltonian

In the quantum electrodynamics model, the field is quantized as shown earlier on in this appendix. $E \rightarrow \hat{E}$, and the interaction Hamiltonian becomes

[20] $\hat{H}_0$ is the unperturbed Hamiltonian.

[21] As defined in Figure 6, $\Delta = \omega_0 - \omega$ is the detuning, which vanishes upon resonance. ω_0 is the atomic transition frequency and ω is the laser frequency.

$H^I = -\vec{d} \cdot \hat{E}$.[22] Chunking out the usual math and making the Rotating Wave Approximation, one arrives at the *Jaynes-Cummings Hamiltonian*,

$$H = \frac{1}{2}\hbar\omega_0\hat{\sigma}_3 + \hbar\omega a^\dagger a + \hbar\lambda\left(\sigma_+ a + \sigma_- a^\dagger\right), \tag{24}$$

where $\lambda = dg/\hbar$ and the aesthetic operators obey the following relations,

$$\sigma_+ = |e\rangle\langle g| \tag{25}$$
$$\sigma_- = |g\rangle\langle e| = \sigma_+^+ \tag{26}$$
$$\sigma_3 = |e\rangle\langle e| - |g\rangle\langle g| \quad \textit{inversion operator}, \tag{27}$$

along with the commutation relations, $[\sigma_+, \sigma_-] = \sigma_3$ and $[\sigma_3, \sigma_\pm] = 2\sigma_\pm$.

If the atom is initially in the ground state $|g\rangle$ and the field is in state $|n\rangle$,[23] then the initial and final states (excited $|e\rangle$) are given by,

$$|i\rangle = |g\rangle|n\rangle \tag{28}$$
$$|f\rangle = |e\rangle|n-1\rangle. \tag{29}$$

Repeating the formalism outlined in the semiclassical section, one arrives at the probability coefficients, and the following atomic inversion function,

$$W(t) = \sin(2\lambda(n+1)^{1/2}t), \tag{30}$$

where $\boxed{\Omega(n, \Delta = 0) = 2\lambda\,(n+1)^{1/2}}$ is the *QED Rabi frequency* for $\Delta = 0$.

Note that *unlike* the semiclassical Rabi model, for the case of 0-field, there is oscillatory interaction with the vacuum modes,

$$n = 0 \Rightarrow W(t) = \sin(2\lambda t), \tag{31}$$

which properly predicts that the atom spontaneously emits a photon, reabsorbs it, re-emits it, and so on in the cycle of reversible spontaneous emission.

B.2.1 Dressed States: AC Stark Shift

Alone by themselves, the unperturbed levels of the atom are *bare*, and thus are called *bare states*. In the presence of a field, they become *dressed states*. Dressed states have the characteristic of hyperfine splitting. Since the (electric) field varies in time, this is akin to an AC Stark Shift. The Rabi frequency for dressed states is given by $\boxed{\Omega_n(\Delta) = \left(\Delta^2 + 4\lambda^2(n+1)\right)^{1/2}}$.[24]

[22] d is still the dipole matrix.
[23] There are, initially, n photons in the field.
[24] Earlier on in this paper, dressed states are better known as polaritons.

"If I have seen further it is by standing on ye shoulders of Giants."

—Isaac Newton

References

[1] C. C. Gerry and P. L. Knight. *Introductory Quantum Optics*, (Cambridge Univ. Press, Cambridge, 2005).

[2] P. Grangier, G. Roger and A. Aspect. *Europhys. Lett.*, **1** (1986), 173.

[3] J. Marangos. *Nature* **397**, 559-560 (1999).

[4] U. Leonhardt *Phys. Rev. A* **65**, 043818 (2002).

[5] S. E. Harris. *Phys. Today* **50**, 36-40 (1997).

[6] M. Fleischhauer and M. D. Lukin. *Phys. Rev. Lett.* **84**, 5094 (2000).

[7] L. Hau, et al., *Europhysics News* **35** 2 (2004).

[8] D. Budker, et al. *Phys. Rev. Lett.* **81** 5788 (1998).

[9] D. F. Phillips, et al., *Phys. Rev. Lett.* **86**, 783-786 (2001); O. Kocharovskaya, et al., *Phys. Rev. Lett.* **86**, 628-631 (2001).

[10] Y. Vlasov, et al. *Nature* **438**, 65-69 (Nov. 3, 2005).